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ANALYTICAL COMPUTATION OF THE THREE-DIMENSIONAL STEADY-STATE TEMPERATURE CONDITION OF A DAM

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Temperature conditions are of very great significance for earth dams based on permafrost ground. One of the indices of the temperature condition of non-filtering earth dams on a permafrost base is the steady-state (temporally invariable) temperature distribution for mean annual conditions, that is, for the mean water level in the reservoir, and the mean temperature readings at ground level. Tasks such as these have been reviewed for two-dimensional conditions on the assumption of a very broad river valley as compared with the

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cross-sectional dimensions of the dam [1, 3]. But consideration also must be given to three-dimensional conditions, where dams abut the sides of valleys, for example, in order to evaluate the cooling influence of the sides of a valley, and other factors, in order to solve engineering problems. These questions ordinarily are elucidated by electrothermal analogy testing of models. These factors are the reason for seeking an analytical solution to the three-dimensional problems of the steady-state temperature field.

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Temperature conditions are of very great significance for earth dams based on permafrost ground. One of the indices of the temperature condition of nonfiltering earth dams on a permafrost base is the steady-state (temporally invariable) temperature distribution for mean annual conditions, that is, for the mean water level in the reservoir, and the mean temperature readings at ground level. Tasks such as these have been reviewed for two-dimensional conditions on the assumption of a very broad river valley as compared with the cross-sectional dimensions of the dam [1, 3]. But consideration also must be given to three-dimensional conditions, where dams abut the sides of valleys, for example, in order to evaluate the cooling influence of the sides of a valley, and other factors, in order to solve engineering problems. These questions ordinarily are elucidated by electrothermal analogy testing of models. These factors are the reason for seeking an analytical solution to the three-dimensional problems of the steady-state temperature field.

The steady-state temperature distribution, U, with no filtration, and no sameness or anisotropism of the ground, can be described by the Laplace equation [5]

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0, \tag{1}$$

where

x, y, z are three coordinates that are independent variables.

The problem of the abutment of a dam to the side of a valley was suggested as a result of the search for uncomplicated three-dimensional boundary conditions that would make it possible to obtain a comparatively simple solution to the Laplace equation. The problem can be stated as follows.

Let us say we have a half-space bounded by a plane, z = 0 (Figure 1a). The coordinate axes OX and OY are located in this plane, and their positive directions represent the water's edge. One direction of the edge extends along the river valley, the other along the dam (across the valley). Accordingly, the quadrant between the positive directions of the OX and OY axes is occupied by the water in the reservoir, and the surface of the ground that is under the water can be assumed to have a temperature value U = 1. The ground surface over the rest of the area is bounded by the air, and this part of the surface can be assumed to have a temperature value U = 0.

The boundary conditions described can be written in the form

$$
\begin{array}{ll}
\text{when } x \geq 0 \text{ and } y \geq 0 & U = 1 \\
\left.\begin{array}{l}\text{when } x \leq 0 \text{ and } -\infty < y < \infty \\ \text{when } x \geq 0 \text{ and } y \leq 0\end{array}\right) & U = 0
\end{array}
$$

A precise analytical solution to the problem described was obtained with the participation of Professor S. A. Gel'fer, Doctor of Physicomathematical Sciences [6]. It has the form

$$U = \frac{1}{4} - \frac{1}{\pi} \operatorname{arc\,tg} \frac{x^2 + y^2 - (x + y)(z + \sqrt{x^2 + y^2 + z^2})}{(z + \sqrt{x^2 + y^2 + z^2})^2 - (x + y)(z + \sqrt{x^2 + y^2 + z^2})} . \qquad (2)$$

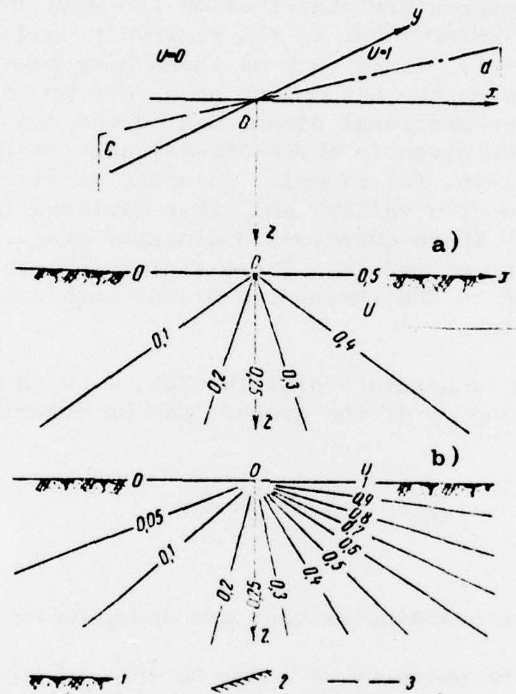

Figure 1. Steady-state temperature distribution at the abutment of a dam to the side of a valley. a - axonometric scheme of coordinate axes, edges, and planes of vertical sections; b - temperature section in the vertical plane, y = 0, passing through the edge; c - temperature section in the vertical plane, x = y, passing through the bisector of the right angle between the sections.

1 - Surface of the ground at the section; 2 - line of the edge in the axonometric representation with the hatching turned toward the reservoir; 3 - trace of the plane of section cd in the axonometric representation.

The isothermal surfaces, that is, the surfaces for which U = constant, are conical surfaces with a common apex in the origin, and two common generatrices that coincide with the positive directions of the coordinate axes OX and OY. Figures 1b and 1c show the vertical temperature sections passing through the origin. The traces of the isothermal surfaces are represented by straight lines in these sections.

This solution does not take into consideration the special features of the relief of the valley, river, or dam, so it can be applied directly for engineering purposes only if the relief is sufficiently flat, and the object is a flattened earth dam.

The problem described can be used to obtain other, equally accurate, solutions to steady-state temperature problems, but for gentle relief conditions. Specifically, by using Eq. (2), and the formula for the two-dimensional problem of the type

$$U = 0{,}5 - \frac{1}{\pi} \operatorname{arc\,tg} \frac{x}{z}, \qquad (3)$$

we can, by using the temperature field summation (superposition) method, obtain an accurate solution to another important engineering problem, that of the steady-state field in the soils of a valley and the soil material in the body of a dam when the width of the reservoir is B and there is no water in the after bay. Figure 2a is the scheme of the arrangement of the coordinate axes. Here, as before, the surface temperature of the ground in contact with the water is taken U = 1, and that in contact with the air U = 0.

The boundary conditions can be written in the form

$$\begin{array}{lll} \text{when } x \le 0 & \text{and } -B/2 \le y \le B/2 & U = 1 \\ \text{when } -\infty < x < \infty & \text{and } -B/2 \ge y \ge B/2 \;) & \\ \text{when } x \ge 0 & \text{and } -\infty < y < \infty \;) & U = 0 \end{array} \qquad (4)$$

A solution can be offered by the formula

$$U = \frac{1}{\pi}\left\{ \operatorname{arc\,tg} \frac{x^2 + (y - 0{,}5B)^2 + (x - y + 0{,}5B)[z + \sqrt{x^2 + (y - 0{,}5B)^2 + z^2}]}{[z + \sqrt{x^2 + (y - 0{,}5B)^2 + z^2}]^2 + (x - y + 0{,}5B)[z + \sqrt{x^2 + (y - 0{,}5B)^2 + z^2}]} + \right.$$

$$\operatorname{arc\,tg} \frac{x^2 + (y + 0{,}5B)^2 + (x + y + 0{,}5B)[z + \sqrt{x^2 + (y + 0{,}5B)^2 + z^2}]}{[z + \sqrt{x^2 + (y + 0{,}5B)^2 + z^2}]^2 + (x + y + 0{,}5B)[z + \sqrt{x^2 + (y + 0{,}5B)^2 + z^2}]} - \qquad (5)$$

$$\left. - \operatorname{arc\,tg} \frac{x}{z} \right\}.$$

Using this solution, we have presented three characteristic vertical temperature sections in Figures 2b, c, and d: through the axis of the reservoir (y = 0); through the edge along the dam (x = 0); and across the reservoir (x = −B). At the same time, the latter two sections are delimited by the plane of symmetry, y = 0. The width of the reservoir, B, has been taken as the unit of length in these sections.

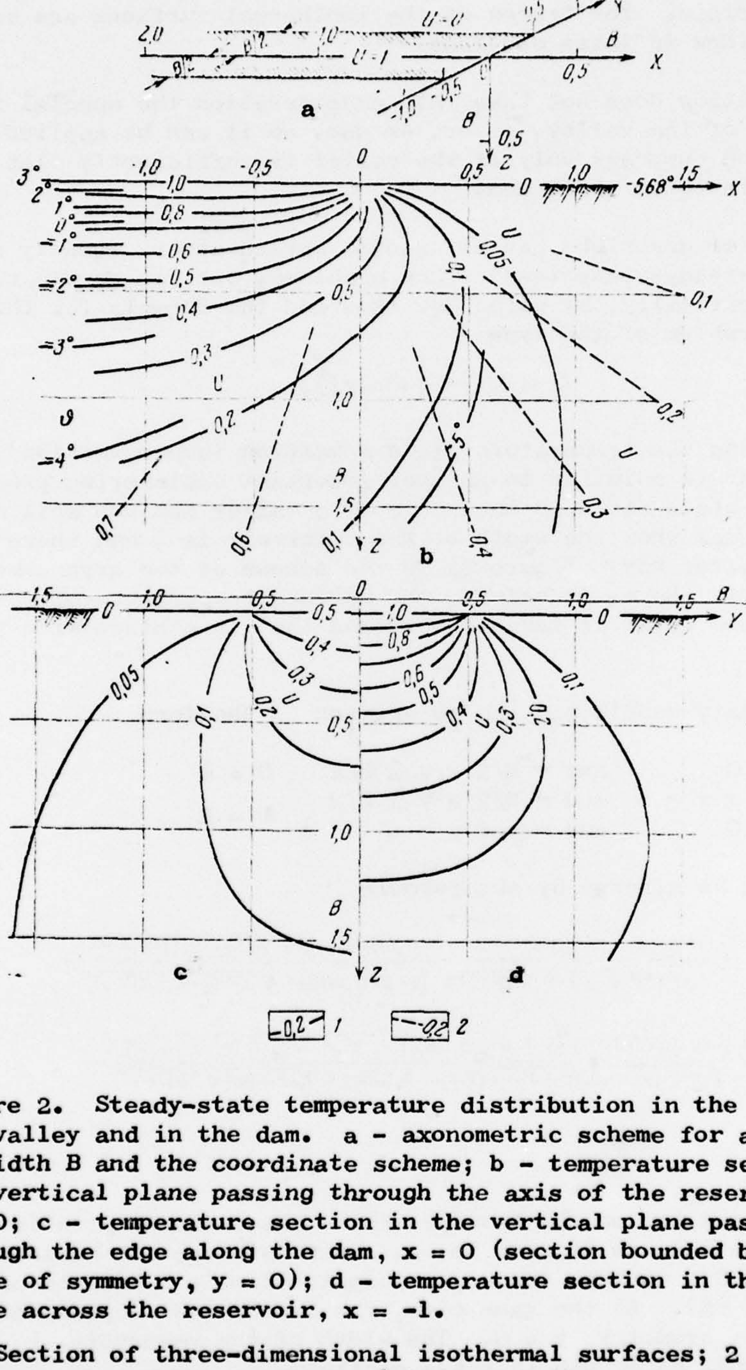

Figure 2. Steady-state temperature distribution in the soil of the valley and in the dam. a - axonometric scheme for a reservoir of width B and the coordinate scheme; b - temperature section in the vertical plane passing through the axis of the reservoir, y = 0; c - temperature section in the vertical plane passing through the edge along the dam, x = 0 (section bounded by the plane of symmetry, y = 0); d - temperature section in the vertical plane across the reservoir, x = -1.

1 - Section of three-dimensional isothermal surfaces; 2 - two-dimensional problem isotherms (sides of valley not taken into consideration).

Several isotherms (straight lines) corresponding to the two-dimensional problem in the case of a very wide reservoir [Eq. (3)], have been plotted in Figure 2b as dashed lines to represent the influence of the banks (the influence of space). The difference in the configuration of the isotherms, constructed with, and without, consideration of three-dimensional conditions, is obvious. The difference will be aggravated as the sides of the valley are approached.

The solutions cited obviously can be used to obtain other solutions with more complex boundary conditions, but without taking the relief of the river valley and dam into consideration. The influence of the heat of the earth's interior can be taken into consideration, as was done in the case of the two-dimensional problem [1], for example.

The formulas cited in the foregoing determine the relative soil temperature, U. Zero on this relative temperature scale is taken as the temperature of the surface of the ground in contact with the air, and the difference in temperatures between that of the underwater surface of the ground and that of the surface in contact with the air is taken as 1. Moreover, the formulas have been derived on the assumption of the invariability of the thermal conductivity coefficient for the ground, λ.

But we must, for practical reasons, present the temperature field in the conventional system, ϑ°C, and take into consideration changes in the thermal conductivity coefficient as a function of temperature

$$\lambda = f(\vartheta). \tag{6}$$

As a practical matter, this dependence is particularly noticeable when the values of the thermal conductivity coefficient for a particular soil in the thawed and frozen states are compared. The steady-state temperature field will be described by a more complex equation than Eq. (1) when Eq. (6) is taken into consideration; namely

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial \vartheta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial \vartheta}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial \vartheta}{\partial z}\right) = 0. \tag{7}$$

This equation can be written in yet another form

$$\frac{d\lambda}{d\vartheta}\left[\left(\frac{\partial \vartheta}{\partial x}\right)^2 + \left(\frac{\partial \vartheta}{\partial y}\right)^2 + \left(\frac{\partial \vartheta}{\partial z}\right)^2\right] + \lambda\left(\frac{\partial^2 \vartheta}{\partial x^2} + \frac{\partial^2 \vartheta}{\partial y^2} + \frac{\partial^2 \vartheta}{\partial z^2}\right) = 0. \tag{8}$$

The same differential equation can be obtained from Eq. (1) if it is taken that U and ϑ are linked by the dependency

$$U = \frac{1}{b}\int_0^{\vartheta} \lambda d\vartheta + c, \tag{9}$$

where

b and c are arbitrary constants.

What follows then is that given identical boundary conditions, the isothermal surfaces determined from Eqs. (1) and (7) will have the same shape. All that remains is to find the quantitative relationship between U and ϑ by finding the constants b and c.

Comparison of the boundary conditions

$$\begin{aligned} &\text{when } U = 0 \qquad \vartheta = \vartheta_{lower} \\ &\text{when } U = 1 \qquad \vartheta = \vartheta_{upper} \end{aligned} \tag{10}$$

establishes the fact that

$$b = \int_{\vartheta_1}^{\vartheta_u} \lambda d\vartheta \quad \text{and} \quad c = \int_{\vartheta_1}^{\vartheta} \lambda d\vartheta. \tag{11}$$

From whence, finally, Eq. (9) will take the form

$$U = \frac{\int_{\vartheta_1}^{\vartheta} \lambda d\vartheta}{\int_{\vartheta_1}^{\vartheta_u} \lambda d\vartheta}. \tag{12}$$

This complex integral equation can be solved comparatively simply by resort to graphical means [1].

This last formula can be simplified if the ground in the frozen state has a constant value for the thermal conductivity coefficient, λ_{fg}, as well as in the thawed state, λ_{tg},

for the zone of frozen ground $\vartheta \leq 0$

$$U = \frac{\lambda_{fg}(\vartheta - \vartheta_1)}{\lambda_{fg}(-\vartheta_1) + \lambda_{tg}\vartheta_u}, \tag{13}$$

in the zone of thawed ground $\vartheta \geq 0$

$$U = \frac{\lambda_{fg}(-\vartheta_1) + \lambda_{tg}\vartheta}{\lambda_{fg}(-\vartheta_1) + \lambda_{tg}\vartheta_u}. \tag{14}$$

If the mean annual temperature of the surface in contact with the air is $\vartheta_1 = -5.68°$, and if the mean annual temperature of the surface in contact with the water in the reservoir is $\vartheta_u = 3.0°$, and if the values of the thermal conductivity coefficients are $\lambda_{fg} = 0.0833$ Mcal/m day degree (3.47 kcal/m hour degree) and $\lambda_{fg} = 0.544$ Mcal/m day degree (2.26 kcal/m

hour degree),* the last two formulas cited can be used to find the numerical dependency between the temperature on the scale of degrees ϑ and the relative temperature U as follows

ϑ °C	−5,68	−5	−4	−3	−2	−1	0	1	2	3
U	0	0,09	0,22	0,35	0,48	0,61	0,74	0,82	0,91	1

The positions of the isotherms, ϑ, in degrees, have been plotted in Figure 2b in accordance with the calculated spacing of the relative isotherms, U.

The accurate solutions described can be used to develop approximate methods for calculating the steady-state temperature condition, while at the same time taking the terrain and dam relief into consideration.

CONCLUSIONS

1. An accurate analytical solution for the three-dimensional steady-state temperature condition of soils in a gentle valley with a reservoir formed by a flattened earth dam has been obtained.

2. The proposed solution is applicable to a nonfiltering dam based on permafrost. The calculation to obtain the different values of the thermal conductivity coefficients for ground in the thawed and frozen states for three-dimensional conditions can be made in accordance with the laws that are applicable to the two-dimensional problem.

3. The solutions cited indicate the practically important temperature conditions that occur in dams; specifically:

(a) the intrinsically great cooling of the dam at its point of contact with the side of the valley as compared to that taking place in the dam's center section;

(b) the considerable effect of the cooling of the dam in its center section originating from the sides of the valley, making it possible to evaluate the narrowness of the cross section as a means of increasing cooling (freezeability) of the dam.

4. The proposed accurate analytical solutions can be used as the basis for finding accurate solutions to other three-dimensional problems with more complex conditions, but without taking the relief of the valley and dam into consideration.

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*Translator's note. Both λ have the same subscripts in Russian. Based on the formulas it would appear that one λ should apply to frozen ground (fg), the other to thawed ground (tg).

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